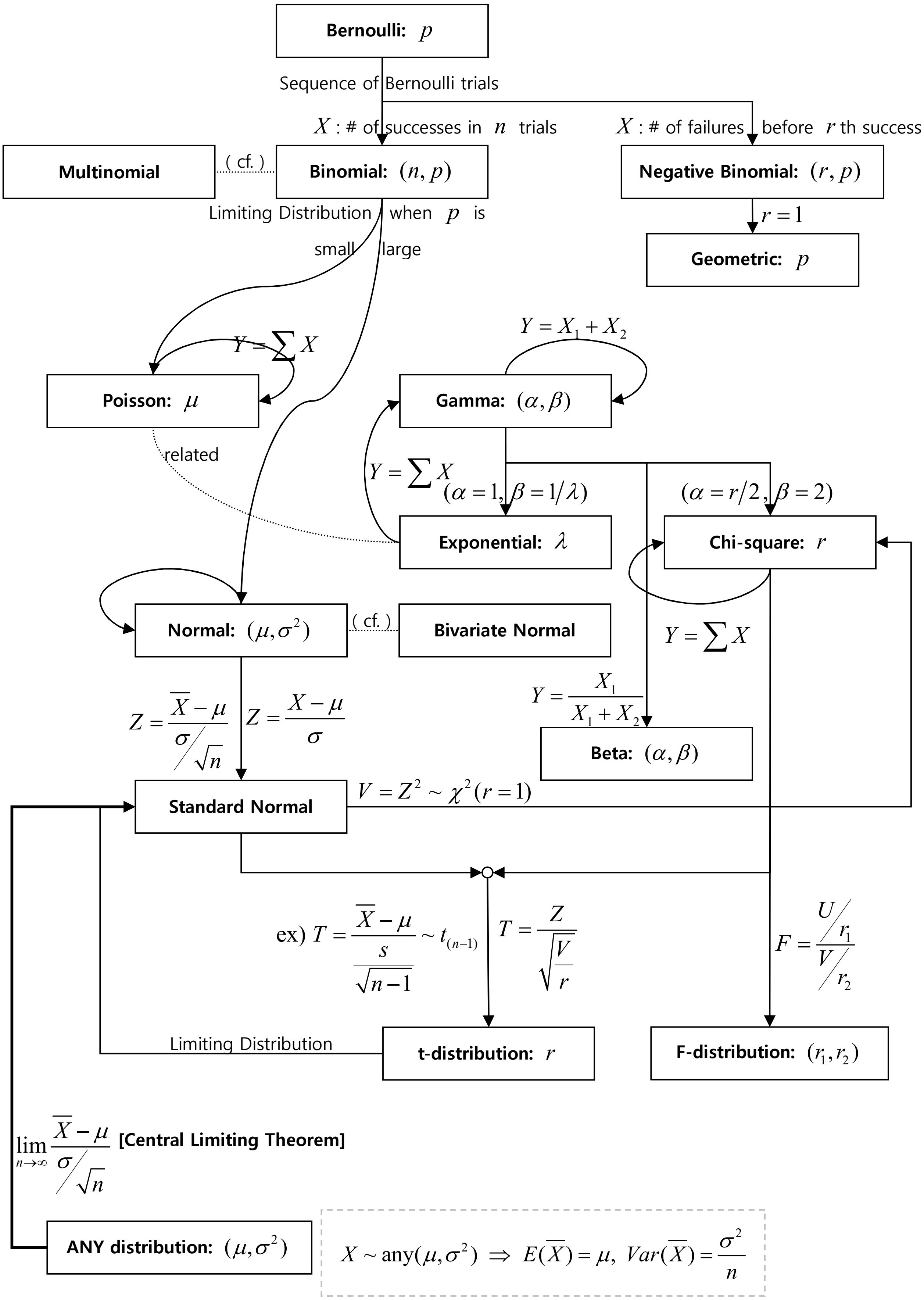
○ 통계학에서 밝혀 둔 다양한 확률분포 중 일부를 정리해 보면, 각각의 확률분포는 서로 독립적으로 존재하는 것이 아니라 다음 그림과 같은 관계에 있다.



○ 앞 그림에 포함된 각 확률분포의 특성은 다음과 같다.

- Bernoulli 분포: $0 \le p \le 1$
  - ▫ 확률변수 $X = \{x \mid x \text{ is 1 for "success" or 0 for "failure" }\}$
  - ▫ X 의 확률분포는 $f(x) = p^x (1-p)^{1-x}$ 이다. 단, $x = 0,1$ 이다.
  - ▫ '적률생성함수(moment-generating function)'는 $M(t) = 1 - p + pe^t$ 이다.
  - ▫ 기대값은 $\mu = M^{'}(0) = p$ 이다. 확률적 분산은 $\sigma^2 = M^{''}(0) - \{M^{'}(0)\}^2 = p(1-p)$ 이다.
- Binomial(이항) 분포: integer n, $0 \le p \le 1$
  - ▫ $X = \{x \mid x \text{ is "number" of successes under } n \text{ trials with Bernoulli trial}\}$
  - ▫ $f(x) = {}_nC_x p^x (1-p)^{n-x}$ where $x = 0,1,2,\cdots,n$
  - ▫ $M(t) = [pe^t + (1-p)]^n$
  - ▫ $\mu = np$, $\sigma^2 = np(1-p)$
- Negative Binomial 분포: integer r, $0 \le p \le 1$
  - ▫ $X = \{x \mid x \text{ is "number" of failures before } r^{th} \text{ success with Bernoulli trial}\}$
  - ▫ $f(x) = {}_{(x+r-1)}C_{(r-1)} p^r (1-p)^x$ where $x = 0,1,2,\cdots,n$
  - ▫ $M(t) = p^r [1-(1-p)e^t]^{-r}$
  - ▫ $\mu = \frac{1-p}{p} r$, $\sigma^2 = \frac{1-p}{p^2} r$
- Geometric 분포: $0 \le p \le 1$ [$r = 1$ in Negative Binomial]
  - ▫ $X = \{x \mid x \text{ is "number" of failures before } 1^{th} \text{ success with Bernoulli trial}\}$
  - ▫ $f(x) = p(1-p)^x$ where $x = 0,1,2,\cdots,n$, and $M(t) = p[1-(1-p)e^t]^{-1}$
  - ▫ $\mu = \frac{1-p}{p}$, $\sigma^2 = \frac{1-p}{p^2}$
- Poisson 분포: μ
  - ▫ $f(x) = \dfrac{\mu^x e^{-\mu}}{x!}$ where $x = 0,1,2,\cdots,n$
  - ▫ $M(t) = e^{\mu(e^t - 1)}$
  - ▫ $\mu = \sigma^2 = \mu$
- Applied Poisson 분포: w, λ [$w \times \lambda$ instead of μ in Poisson]
  - ▫ $X = \{x \mid x \text{ is "number" of changes(accidents/claims) in a fixed interval(time/space) }\}$
  - ▫ $f(x, w) = \dfrac{(\lambda w)^x e^{-\lambda w}}{x!}$ where $x = 1,2,\cdots,n$, and $M(t) = e^{\lambda w(e^t - 1)}$
    - · w: a fixed interval, λ: probability of x in w, $\lambda \times w$: mean number of x in w
  - ▫ $\mu = \sigma^2 = \lambda w$

- Gamma 분포: $\alpha > 0, \beta > 0$
  - $f(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta}$ where $0 < x < \infty$
  - $M(t) = [1 - \beta t]^{-\alpha}$
  - $\mu = \alpha\beta$, $\sigma^2 = \alpha\beta^2$
- Applied Gamma 분포: λ, k [$(k, 1/\lambda)$ instead of (α, β) in Gamma]
  - $W = \{w \mid w \text{ is "waiting time" for } k^{th} \text{ under Applied Poisson }\}$
  - $f(w) = \frac{\lambda^k}{\Gamma(k)} w^{k-1} e^{-\lambda w}$ where $0 < w < \infty$
  - $M(t) = [1 - t/\lambda]^{-k}$
  - $\mu = k/\lambda$, $\sigma^2 = k/\lambda^2$
- Exponential 분포: λ [$k = 1$ in Applied Gamma]
  - $W = \{w \mid w \text{ is "waiting time" for } 1^{th} \text{ under Applied Poisson }\}$
  - $f(w) = \lambda e^{-\lambda w}$ where $0 < w < \infty$
  - $M(t) = [1 - t/\lambda]^{-1}$
  - $\mu = 1/\lambda$, $\sigma^2 = 1/\lambda^2$
- Beta 분포: $\alpha > 0, \beta > 0$ [$\frac{\text{Gamma}(\alpha,1)}{\text{Gamma}(\alpha,1)+\text{Gamma}(\beta,1)}$]
  - $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ where $0 < x < 1$
  - $\mu = \frac{\alpha}{\alpha+\beta}$, $\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$
- Chi-square 분포: r [$(r/2, 2)$ instead of (α, β) in Gamma]
  - $f(x) = \frac{1}{\Gamma(r/2)2^{r/2}} x^{(r-2)/2} e^{-x/2}$ where $0 < x < \infty$
  - $M(t) = [1 - 2t]^{-r/2}$ $(t < \frac{1}{2})$
  - $\mu = r$, $\sigma^2 = 2r$
- F 분포: r_1, r_2 [$\frac{\text{Chi-square}(r_1)}{\text{Chi-square}(r_2)}$]
  - $f(x) = \frac{\Gamma((r_1+r_2)/2)}{\Gamma(r_1/2)\Gamma(r_2/2)} (\frac{r_1}{r_2})^{r_1/2} \frac{x^{(r_1-2)/2}}{(1+\frac{r_1}{r_2}x)^{(r_1+r_2)/2}}$ where $0 < x < \infty$
  - $\mu = \frac{r_2}{r_2-2}$, $\sigma^2 = \frac{2{r_2}^2(r_1+r_2-2)}{r_1(r_2-2)^2(r_2-4)}$

- Normal(정규) 분포: μ, σ^2
  - $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{(x-\mu)^2}{2\sigma^2}]$ where $-\infty < x < \infty$
  - $M(t) = \exp[\mu t + (\sigma t)^2 / 2]$, $\mu = \mu$, $\sigma^2 = \sigma^2$
- Standard Normal(표준정규) 분포: [$(0,1)$ instead of (μ, σ^2) in Normal]
  - $f(z) = \frac{1}{\sqrt{2\pi}} \exp[-\frac{z^2}{2}]$ where $-\infty < z = \frac{x-\mu}{\sigma} < \infty$
  - $M(t) = \exp[t^2 / 2]$, $\mu = 0$, $\sigma^2 = 1$
- t 분포: r [$\frac{\text{Standard Normal}}{\text{Chi-square}(r)/r}$]
  - $f(t) = \frac{\Gamma((r+1)/2)}{\sqrt{2r}\,\Gamma(r/2)} \frac{1}{(1+t^2/r)^{(r+1)/2}}$ where $-\infty < t < \infty$
  - $t = \frac{z}{\sqrt{v/r}}$ [$z \in Z \sim N(0,1)$ and $v \in V \sim \chi^2(r)$ where $\chi^2(r)$ is Chi-square]
  - $\mu = 0$, $\sigma^2 = \frac{r}{r-2}$ (only when $r > 2$)